Assessing properties of *E. nitens* laminated flooring

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EXECUTIVE SUMMARY

Worldwide there is increased interest in engineered flooring products, which combine the look of solid wood with increased dimensional stability (i.e. reduced movement in service) and reduced distortion. Specialty Timber Solutions in North Canterbury produce flooring from pruned eucalypts, including *Eucalyptus nitens*, and have developed two engineered flooring products from *E. nitens*. The aim of this study was to compare these new products with solid *E. nitens* flooring, as well as commercially available engineered flooring (European oak with a birch plywood backing), to assess their suitability for an overlay flooring product (either fixed or floating installation). Additionally it was decided to use unpruned pulp regime *E. nitens* to produce the flooring, as this represents the majority of the *E. nitens* resource in New Zealand.

Specialty Timber Solutions produced three kinds of flooring from unpruned *E. nitens* logs – solid, cross-laminated and birch plywood-backed flooring. The performance of these was compared to commercial engineered oak flooring by looking at the board dimensions and levels of distortion at several different ambient humidity levels, and also while soaking one face in water.

When the board moisture content changed, the solid wood flooring changed width significantly more than all the other flooring types. Changes in board width are a major cause of movement-related issues with flooring, so the increased stability of the engineered boards is a significant result. At low moisture content the cross-laminated flooring cupped significantly more than the other flooring types. Cupping tends to only be a problem with floating overlay floors where the board edges are not restrained, and in this situation, it is possible that wide cross laminated boards would cup to an unacceptable extent. Otherwise the dimensions and distortion of the different flooring types were all similar when exposed to changes in air humidity or soaked in water.

The different types of *E. nitens* flooring all had a similar surface hardness (average ~4kN), which was lower than the engineered oak flooring (average ~5.5kN). This would potentially make the *E. nitens* flooring more susceptible to damage in service and may therefore make it a lower value product.

The cross laminated *E. nitens* flooring had the lowest strength and stiffness in third-point bending but was similar to one batch of the engineered oak boards which had a grooved plywood backing (because it is designed as an overlay flooring product). This suggests that the cross-laminated boards are suitable for overlay flooring as intended. The ply-backed and solid wood *E. nitens* boards had significantly higher strength and stiffness than the engineered oak.

Overall the differences between the engineered oak and *E. nitens* boards were quite small. The lower hardness of the *E. nitens* boards may be a disadvantage compared to the oak. In situations where changes in board width need to be minimised, the ply-backed or cross laminated *E. nitens* were more dimensionally stable than the solid *E. nitens*, but the cross-laminated boards are more prone to cupping, which may cause issues in service unless adequately glued and nailed to the floor substrate.

INTRODUCTION

Hardwood flooring is popular as a premium flooring product and is a common end use for non-durable eucalypts in Australia. Increasingly, engineered wood flooring products are being developed to give a product with the look and feel of wood, but with improved stability (both dimensional stability, and resistance to distortion e.g. cupping).

John Fairweather at Specialty Timber Solutions has started producing two laminated flooring products from pruned *Eucalyptus nitens* as additional products to the solid hardwood flooring he already produces. One product is a two-layer cross-laminated board, and the other has a plywood backing and a single 8mm thick lamella of *E. nitens*. These products have several advantages over producing flooring from a single thicker board:

- thinner boards are easier to dry without checking and collapse;
- boards can be flatsawn, lowering costs of production and improving sawn recoveries with wider boards sawn from smaller logs; and
- dimensional stability should be improved as the bottom layer is oriented perpendicular to the top layer.

Flooring can be installed either as strip flooring over joists or battens, or as an overlay which is installed over a solid substrate such as concrete or plywood. Some of the flooring products assessed here (cross-laminated *E. nitens* and 15mm thick engineered oak) are only designed to be used as overlay flooring. In the interests of making fair comparisons between the different products, all the products assessed here have only been assessed for their suitability as overlay flooring i.e. load bearing structural performance is not required. Overlay flooring can be installed in a number of ways – boards can be nailed or glued to the substrate or can be floating (boards are connected to each other, but not to the substrate).

This work assessed the performance of the laminated flooring relative to an equivalent solid wood floor product, as well as a competing commercial product (engineered oak). The main objective was to determine if the laminated *E. nitens* flooring has improved dimensional stability over either the existing *E. nitens* solid wood flooring or the engineered oak. Hardness was tested for each flooring type, because this is a key property for flooring performance. Boards were also tested for bending strength and stiffness, to ensure all the boards have sufficient strength to perform in service (while overlay flooring is not a structural, or load bearing application, floor boards will need to have some degree of strength to not break during installation and use).

In 2017/18 John produced all the *E. nitens* floorboards required for this testing from unpruned trees supplied by SouthWood exports (typical harvest age 18-19 years). This was intended to ensure that the properties of the wood were representative of the unpruned *E. nitens* pulp resource in New Zealand. Engineered European white oak flooring was purchased from Timspec in Auckland.

METHODS

Trees for this trial were from an unpruned, unthinned stand of *E. nitens* grown in Southland for pulp wood. For this study 4 trees were selected, and two 6.5m logs were cut from each tree (minimum SED 150mm). The logs were transported to Specialty Timber Solutions in Sefton, North Canterbury to be milled. The logs were cut to 3m lengths prior to milling and were sawn into a mix of 25mm thick quarter sawn boards for solid wood flooring, and 10mm thick flat sawn boards for the engineered *E. nitens* flooring.

The nitens flooring was produced by milling 3m long logs to produce largely quarter sawn boards using the Satchel/Legler sawing method. The boards coming off the saw were typically crooked so they were then edged to make straight boards. Following edging the boards were slowly air dried for 3-6 months by wrapping the pallets with four layers of microclima cloth (frost cloth). When the boards were below 30% MC they were then dried to 12%MC in a solar kiln. The kiln dried boards were made into tongue and groove flooring using a Logosol planer/moulder. This process involved a first pass to dress four side the boards and then a second pass to make the flooring profile. The solid wood flooring has relief grooves along the length of the back face as this is typical for this type of flooring. It is possible that this profile limits the formation of cup relative to an ungrooved solid piece of wood, but because these grooves are typical of solid wood floor boards, it was decided to include them, to give a representative flooring product.

The completed 2.4m long boards were shipped to Scion to be cut into samples for testing. It was originally intended to have thirty 1m long boards per treatment, with all boards with a cover width close to 180mm wide (to enable easy comparisons of cup measurements), but there was insufficient usable sawn timber to produce this. The board length was reduced to 0.8m to allow for 3 boards to be cut from each 2.4m length of ply-backed flooring, to maximise the number of samples tested. Additionally, the solid wood boards were supplied in two different widths - 132mm and 110mm. The details of the boards cut for testing are given in Table 1.

Table 1. Boards cut for testing

Type	Cover width*	Thickness	# boards tested	Lamella thickness
Solid wood	132	19	24	-
Solid wood	110	19	6	-
Cross-laminated	170	14	32	7
Ply-backed	170	19	30	7
Engineered oak	180	20	20	6
Engineered oak	180	15	12	4

* the width of the exposed top surface of each board.

Three bundles of engineered oak flooring were purchased from Timspec in Auckland (6mm oak lamella with birch plywood backing). One bundle was purchased separately from the others and was supplied in a different thickness (15mm compared to 20mm for the first two bundles). Additionally, the ply backing of these thinner boards was grooved across the back face, almost completely through the plywood layer (Figure 10). Images of each flooring type are shown in Figure 1. The 15mm engineered oak is designed to be installed as a fixed overlay floor, but the 20mm engineered oak can be installed either as a fixed or floating overlay floor.

Cup measurements are dependent on the board width and needed to be corrected for the different board widths tested. Details of this correction are in the appendix.

Figure 1. Representative images of each flooring type

Because the trees from SouthWood Exports were unpruned, the flooring timber product contained a moderate number of knots. Most bark encased knots were filled with epoxy filler, and had a smooth surface, but in the solid wood flooring the knots were not filled, and where possible, rough bark encased knots were excluded from the testing.

Stability testing

Each board was end coated with a two-pot epoxy paint, to prevent moisture movement through the end grain of the board and simulate the behaviour of a longer board.

Along the centreline of the upper face of each board, one mark was made half way along the board, and two further marks made 200mm from each end. These show where cupping and board dimensions were measured.

Boards were equilibrated at 25°C 65% RH until their weight stabilised. Cupping was measured at the marked points on each board using a digital dial gauge, and the thickness and width of each board measured at the same points. Bow along the length of the board was measured with a digital dial gauge.

Boards were equilibrated at two further humidity conditions (25°C, 90% RH and 25°C 35% RH) and the dimension and distortion measurements repeated at each condition.

Following conditioning at 35% RH, boards were placed face up on a wet surface (carpet tiles just submerged in water) and the level of cup and bow measured every 15 minutes for two hours. Following water soaking, the boards were air dried in the laboratory for two weeks, then conditioned under standard conditions (20°C, 65% RH) until their weight had stabilised. After conditioning, levels of cup and bow were measured on each board one final time before mechanical testing. This was to understand if there were any long-term changes to the shape of the boards following water soaking.

There are no specific performance standards for dimensional stability and distortion in flooring, so this testing was done on a comparative basis – if the *E. nitens* engineered products performed as well as, or better than, the *E. nitens* solid wood and commercial engineered oak, they would be assumed to perform well in service. Additionally distortion measurements were compared to AS 2796.1 (Standards Australia, 2006) which gives specifications for distortion in overlay strip flooring at the time of installation. This standard specifies flooring needs to be between 9 and 14% moisture content. At the high and low humidity conditions used here, wood is normally outside this moisture content range, meaning that the AS 2796.1 does not strictly apply, but it does give an indication of levels of distortion that may become problematic in service.

Mechanical testing

Following dimensional stability testing the boards were equilibrated at 20°C, 65% RH until their weight stabilised. They were then tested in four point bending (380mm span) and following bending, Janka hardness was measured on the top face of each board. This was a comparative test to see how the strength and hardness varied between the different products.

RESULTS

Stability testing

Changes in board width can have a significant impact on the performance of a floor, either through swelling and buckling of the boards, or through shrinkage and gaps opening up between boards. The change in width of the boards between conditioning at 35% and 90% relative humidity are shown in Figure 1. The solid *E. nitens* boards had a significantly larger change in width compared to all the other flooring types, which were no different to each other. This highlights one of the main advantages of engineered flooring, and it is a positive result that the *E. nitens* products performed similarly to the engineered oak.

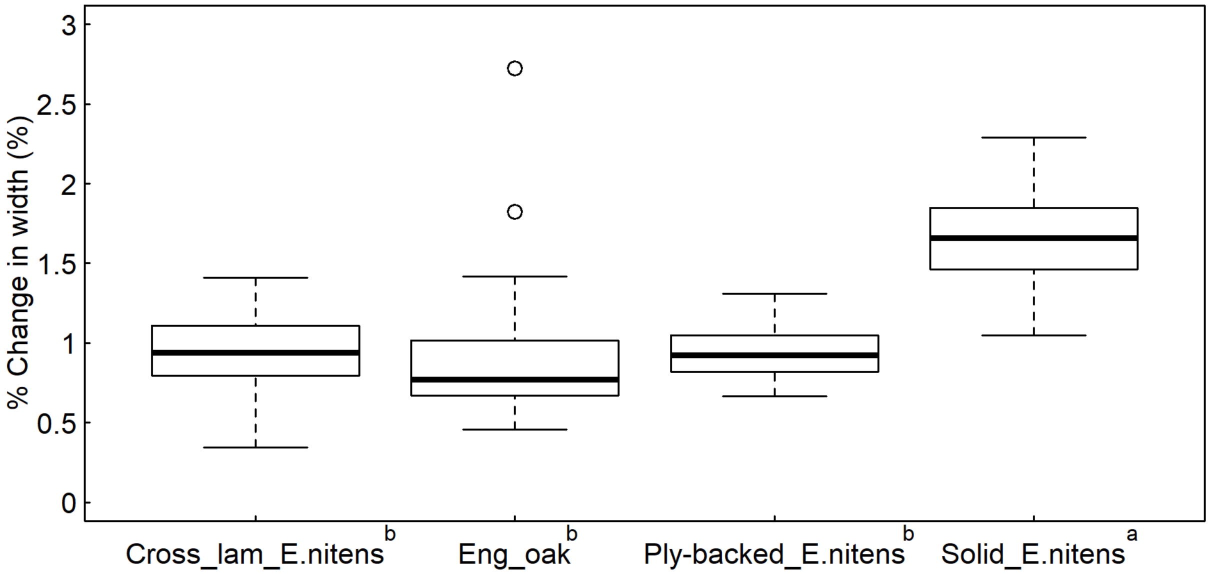

Figure 1. Change in width between 35 and 90% RH conditions for each board type. Superscript letters indicate groups that are not significantly different to each other (95% confidence level)

Figure 2 shows the change in board thickness between conditioning at 35% relative humidity and 90% relative humidity. Changes in board thickness are unlikely to have much impact on the performance of a floor, but as this is primarily a test of comparative performance of different flooring types, comparing changes in board thickness gives a more complete picture of the board behaviour under varying moisture contents. While some of the differences between board types is statistically significant, overall the differences were small.

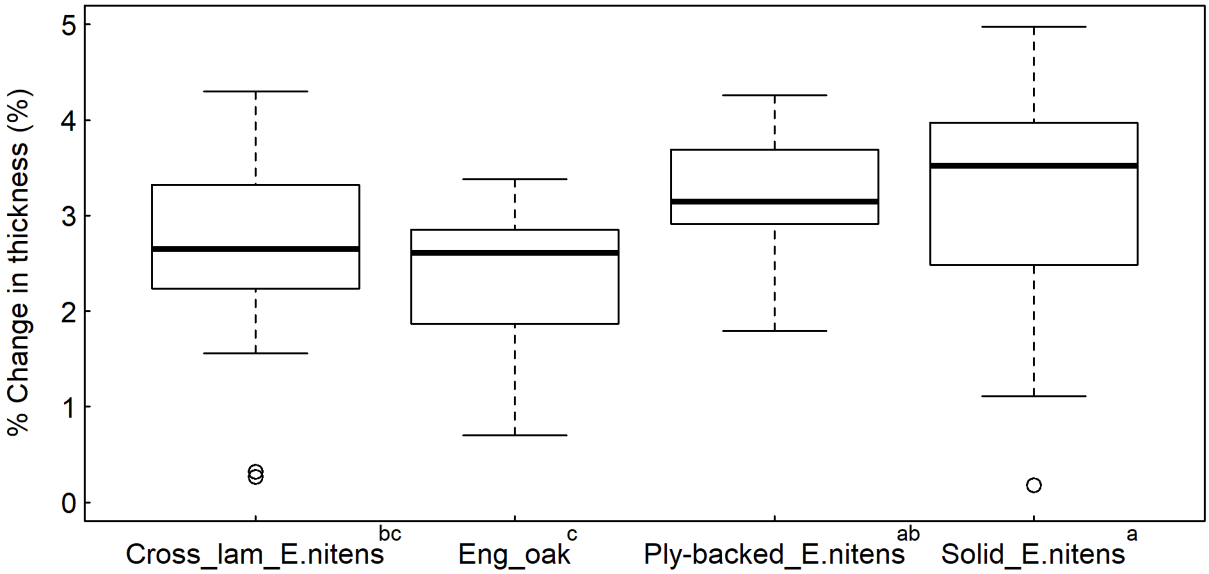

Figure 2. Change in thickness between 35 and 90% RH conditions for each board type. Superscript letters indicate groups that are not significantly different to each other (95% confidence level)

The levels of cup following conditioning at each humidity level are shown in Figure 3. For a fixed overlay floor cupping is not an issue, because the boards are restrained by the substrate, but with wide boards installed as a floating floor cupping could be an issue. For all flooring types at both medium (65%) and high (90%) humidity conditions the levels of cup were quite low and are well below the maximum levels of cup allowed in AS 2796.1 (Standards Australia, 2006) for overlay strip flooring (measured at the time of installation). At the lowest humidity, it is not surprising that overall levels of cup were much higher than at the higher humidities (boards tend to distort more as they become drier). The solid wood boards cupped the least at the low humidity, which is not surprising as they are predominantly quarter-sawn, so are not prone to cupping in the way that the thinner flat-sawn boards would be. The ply-backed *E. nitens* and Engineered Oak cupped a similar amount and were only slightly above the maximum cupping allowed in the standards. One of the engineered oak products tested can be installed as a floating overlay floor, so it is assumed that the cupping in the engineered oak and ply-backed *E. nitens* boards is not sufficient to cause problems in service. The cross-laminated *E. nitens* cupped substantially more than the other flooring types with around twice the level of cupping as the ply-backed *E. nitens*. Cupping is predominantly an issue for wide boards installed as a floating floor. The cross-laminated flooring is intended for producing wider floor boards, so the potential for problematic levels of cupping needs to be taken into account.

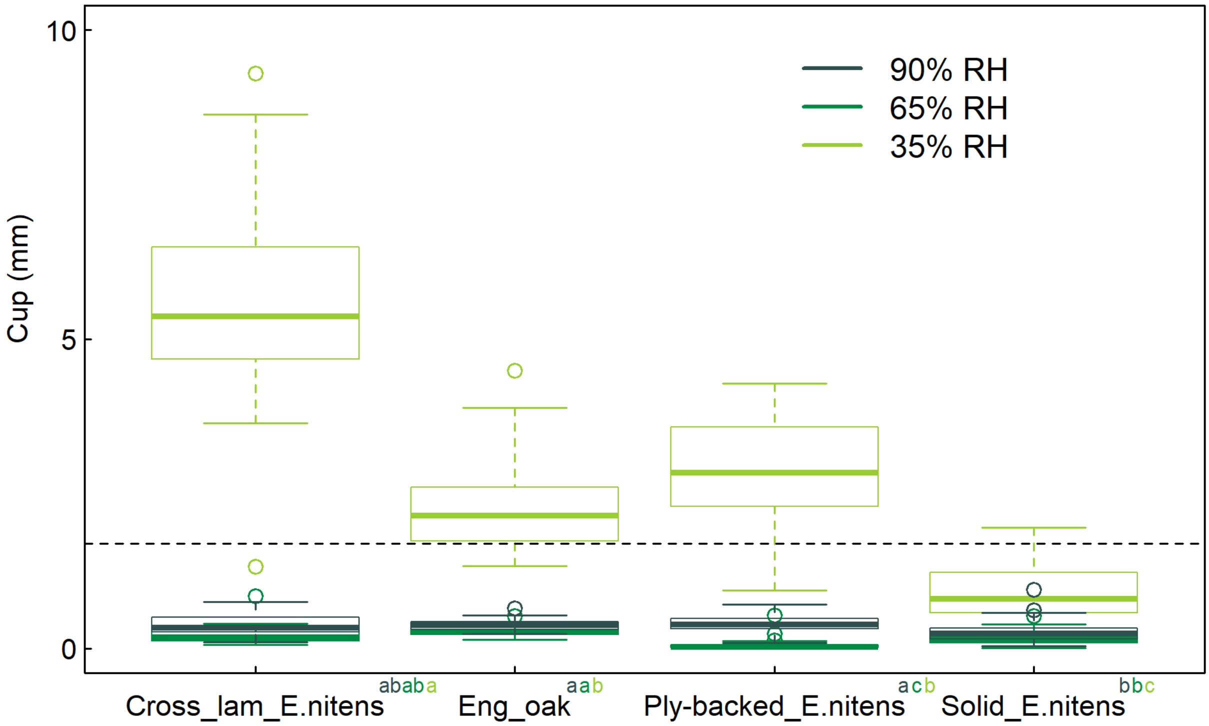

Figure 3. Levels of cup at each moisture content for each board type. Superscript letters indicate groups that are not significantly different to each other (95% confidence level) the text colour of the letters indicates the humidity condition they apply to (as per the figure legend). The dotted line indicates the maximum level of cupping allowed by AS 2796 *at the time of installation*.

Figure 4 shows levels of bow after boards were conditioned at each humidity level. As with cupping, bow is unlikely to be an issue with fixed overlay flooring, but may be a problem with floating floors, although bow is not seen to be as great an issue in flooring as cup, as it is restrained much more easily, so is unlikely to cause significant distortion once boards are connected together. For the solid *E. nitens*, levels of bow were always below the levels allowed in the Australian Standard. All the engineered flooring types (oak and *E. nitens*) had a proportion of boards that were above the levels of bow allowed in the Australian standard when equilibrated at 65% RH, varying from 35% of the engineered oak boards, to 15% of the cross-laminated boards. At the low and high humidity levels the engineered oak and ply-backed *E. nitens* had similar proportions of boards that were above levels allowed by the standards. The cross laminated *E. nitens* had low levels of bow at 65% RH (almost all within levels allowed by the standards) but at low and high humidity levels the majority of the boards had levels of bow that were above those allowed by the Australian standard.

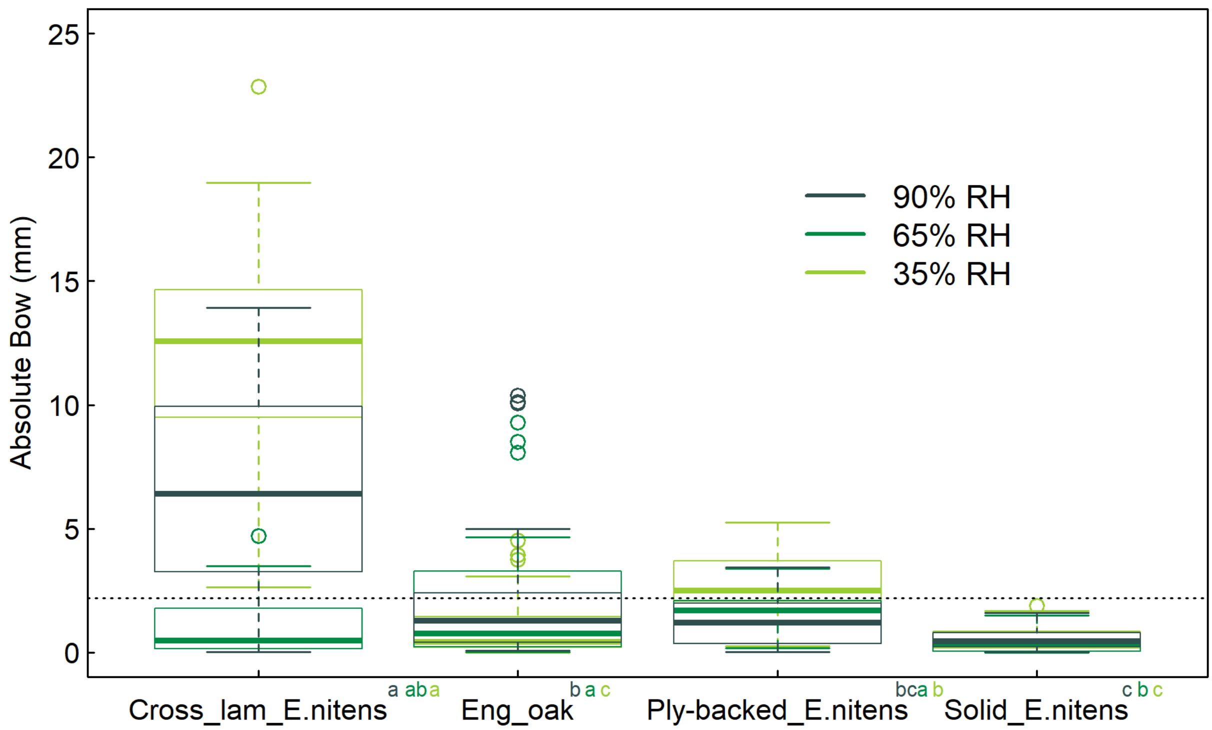

Figure 4. Levels of bow at each moisture content for each board type. Superscript letters indicate groups that are not significantly different to each other (95% confidence level) the text colour of the letters indicates the humidity condition they apply to (as per the figure legend). The dotted line shows the maximum bow allowed in the Australian Standard *at the time of installation.*

Following conditioning at different moisture contents, the boards were placed face up in a shallow puddle of water (~1/2 the thickness of each board) and their cup measured every 15 minutes for two hours. The average cup values for each board type are shown over time in Figure 5. Initially there were large differences between the wood types, with the solid wood boards cupping much less than the other boards. Surprisingly the engineered oak boards initially cupped much more than the other board types. By the end of the 2-hour soaking, the differences between the four board types were very small. Overall the increases in cup were quite small – an average increase of around 0.3mm of cup for each board.

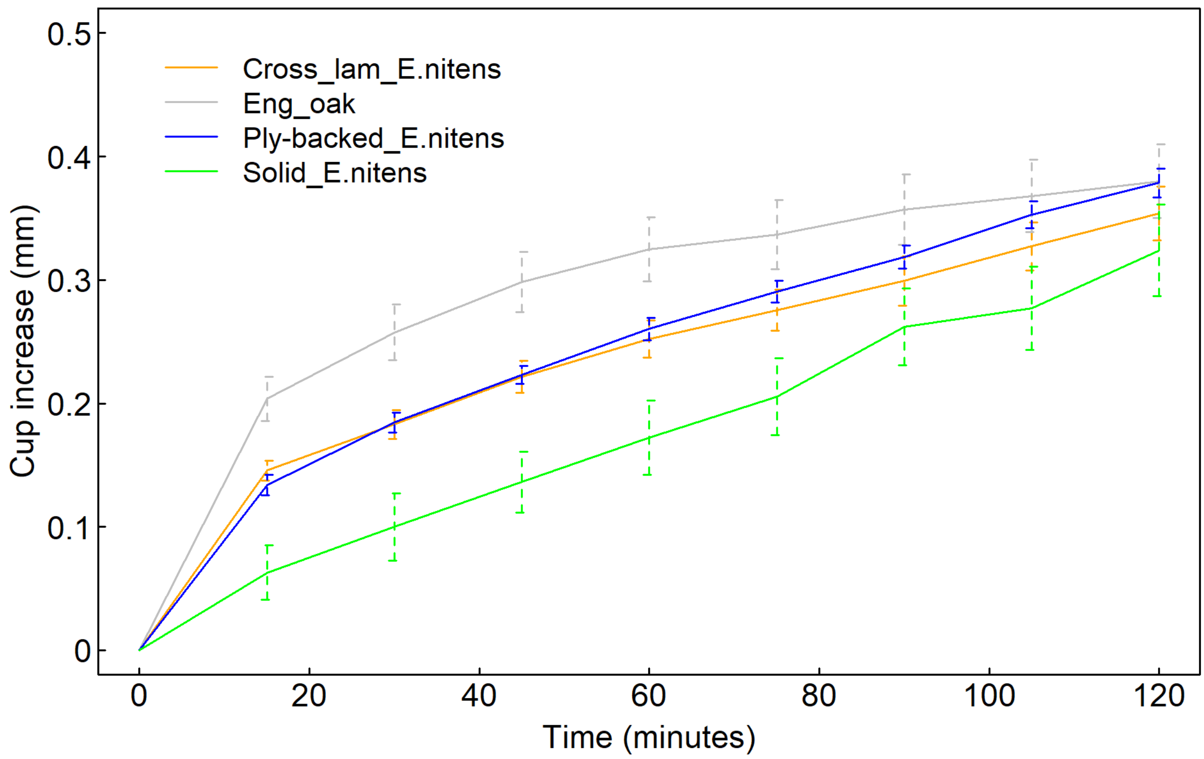

Figure 5. Increases in cup over time for each board type. Error bars indicate one standard error.

Some of the solid wood boards had large knots, and these often wicked water to the board surface during the water soaking (Figure 6). Some engineered boards delaminated between the top face and the substrate during soaking (Figure 7). Approximately 20% of Engineered oak, and cross laminated boards were affected. Pictured is an engineered oak board. No ply-backed boards had visibly delaminated.

Figure 6. Water wicking through a knot to the top face of a solid *E. nitens* board after the lower face was submerged in water

Figure 7. Delamination in an engineered oak board during water soaking

Following water soaking, the boards were conditioned again at 20°C, 65% RH. Following conditioning, levels of cup were not significantly different to at the start of testing, but levels of bow were very slightly higher (average 0.75mm higher).

Mechanical testing and hardness

The hardness values for each flooring type are shown in Figure 8. Not surprisingly the engineered oak had the highest average hardness. Both the cross-laminated and ply-backed *E. nitens* were (statistically speaking) significantly harder than the solid *E. nitens* boards, but not as hard as the engineered oak. The different sawing orientation of the quarter-sawn solid wood boards, and the flat-sawn cross-laminated and ply-backed boards is not expected to have an effect on the hardness values. Hardness testing is sensitive to the substrate below the area that is being tested, so it is likely that the hardness of the birch plywood backing would contribute to the surface hardness of the ply-backed *E. nitens* boards. There are no standards for minimum hardness levels for flooring, but oak is seen to be a good benchmark, so it is possible that a floor made from the laminated *E. nitens* products would dent and wear faster than an oak floor.

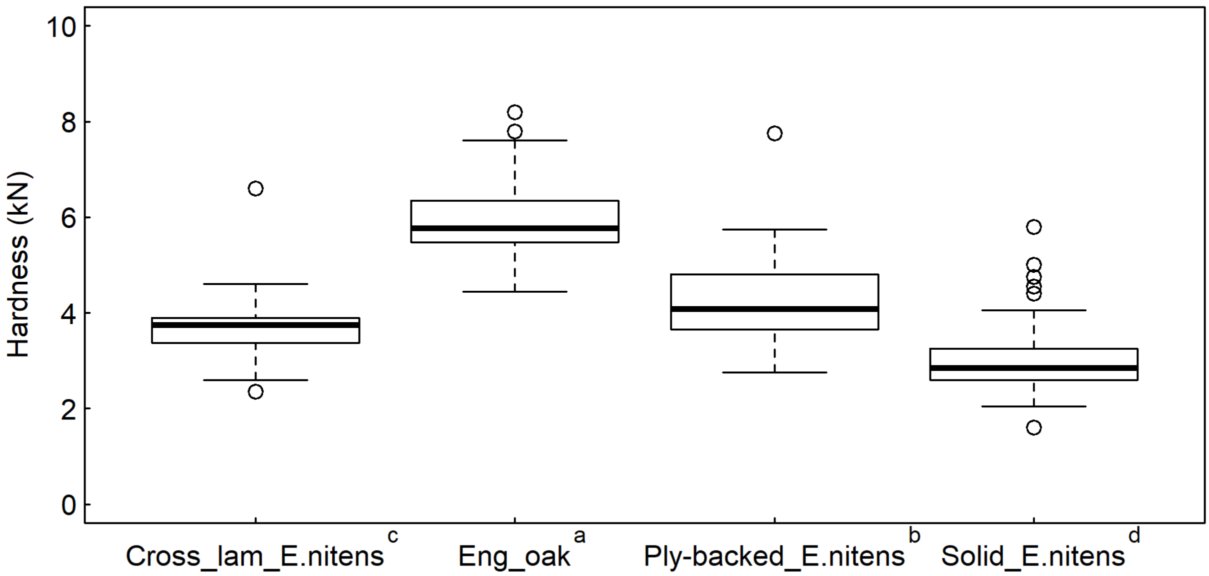

Figure 8. Hardness of the top face of each flooring type. Superscript letters indicate treatments that are not significantly different (95% confidence level).

The engineered oak boards were supplied in two different thicknesses, and on the thinner boards (the overlay product) grooves were cut through the plywood, giving a similar appearance to the cross-laminated *E. nitens* (Figure 10). The stiffness of the different types of flooring is shown in Figure 9, with the thick and the thin engineered oak board shown separately. The thin boards had a significantly lower stiffness than the thicker oak boards, but are obviously sufficiently stiff for their intended use (as an overlay floor). Both sets of engineered oak boards had a lower stiffness than the solid wood or ply backed boards. The cross laminated boards had the lowest stiffness, but again this is an overlay product so high stiffness is not a performance requirement.

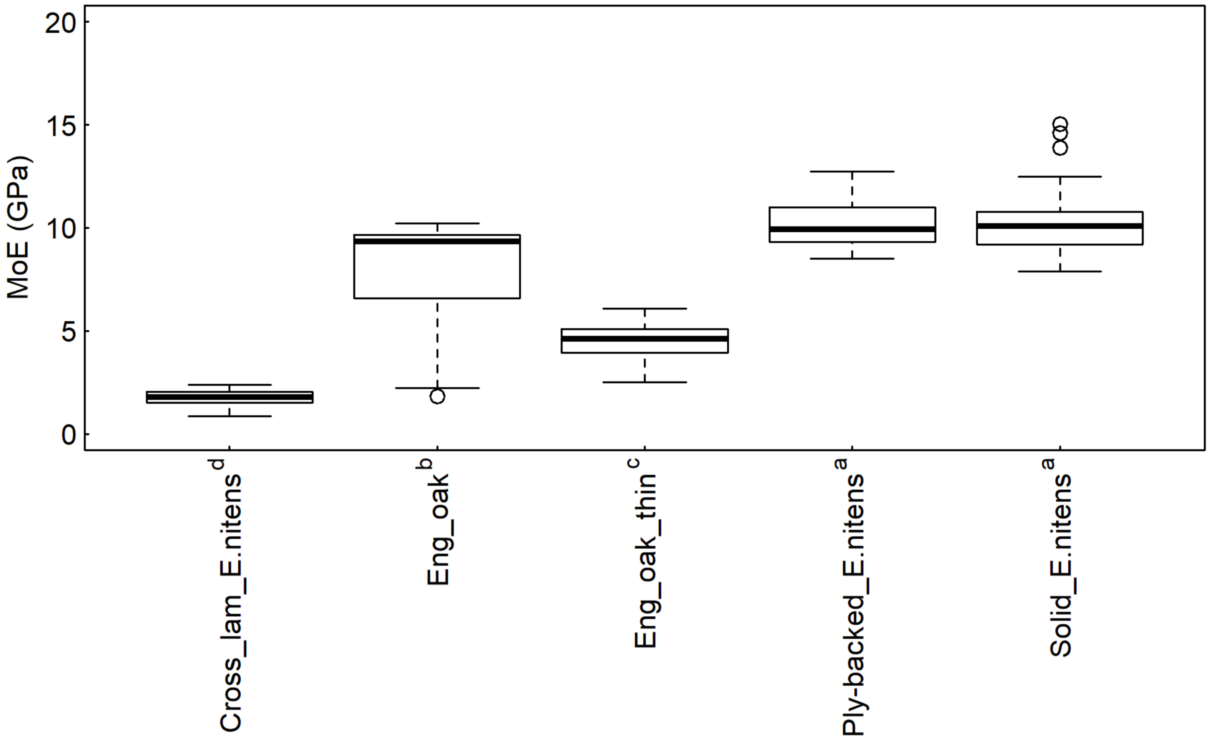

Figure 9. Modulus of elasticity (MOE) of different flooring types. Superscript letters indicate treatments that are not significantly different (95% confidence level).

Figure 10. Back faces of the thin engineered oak (top) and cross-laminated E. nitens (bottom). Both views are of the board edge.

The MOR (Strength) of the different wood types are shown in Figure 11. The thin oak boards are significantly less strong than the thicker oak boards but are not significantly different to the cross-laminated *E. nitens,* which are both overlay flooring products not requiring a high degree of strength. The solid wood, and ply-backed *E. nitens* are significantly stronger than the thicker engineered oak boards. Because the thin engineered oak is a commercial product, it is assumed that it has sufficient strength to perform well as an overlay floor, therefore the cross-laminated boards should also have sufficient strength to perform as overlay flooring.

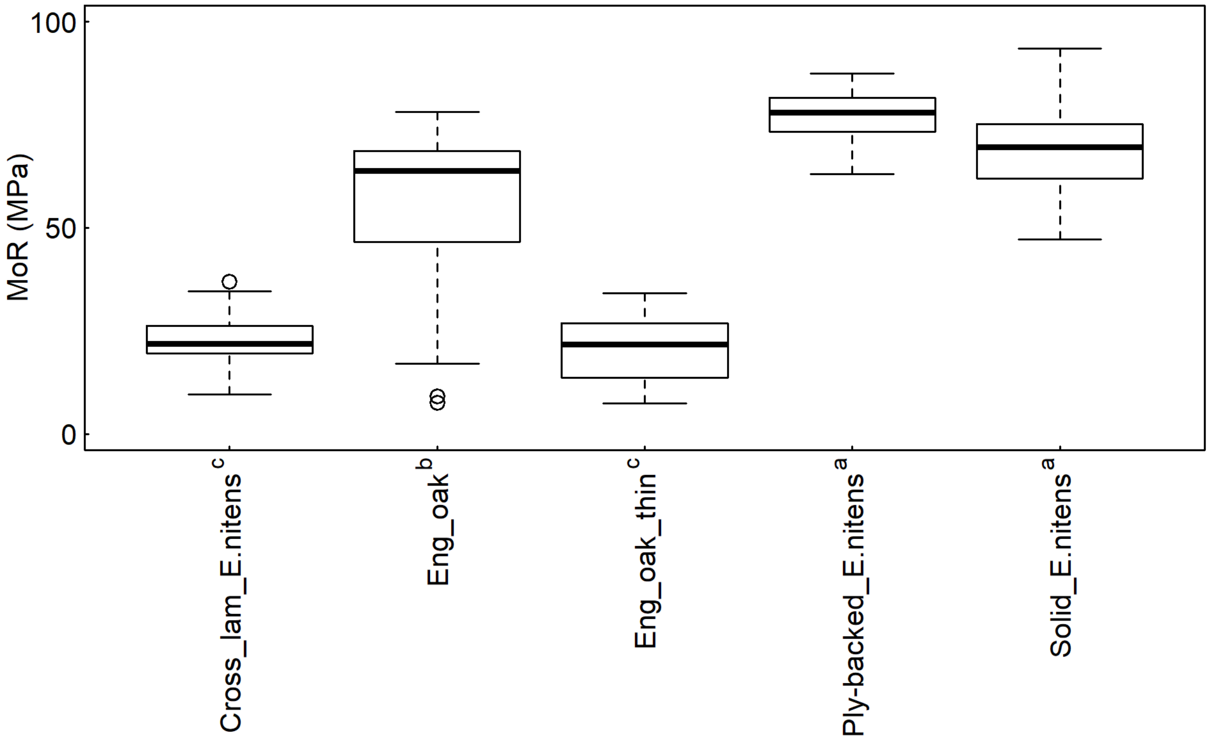

Figure 11. Modulus of rupture (MOR) of different flooring types. Superscript letters indicate treatments that are not significantly different (95% confidence level).

CONCLUSION

The aim of this study was to assess the comparative performance of two novel flooring types (ply-backed and cross-laminated *E. nitens*) against two products already in use for flooring (engineered oak and solid *E. nitens* flooring).

When conditioned at different humidity levels the solid wood boards changed width significantly more than the other flooring types, while the cross-laminated and ply-backed boards performed similarly to the engineered oak. Changes in width are a major cause of problems with flooring, so the engineered *E. nitens* having smaller changes that the solid wood and being similar to the engineered oak is a significant result. Changes in board thickness between low and high humidity conditions were similar between the different wood types.

Distortion (bow and cup) are not an issue with fixed overlay floors, but cupping can be a problem with wide boards installed as floating overlay floors. Overall the solid wood boards had low levels of cupping even when conditioned to high or low moisture contents. The cross-laminated boards had high levels of cupping at low moisture contents. The ply backed *E. nitens* boards performed similarly to the engineered oak boards – these had more cupping that the solid boards but less than the cross-laminated boards. Additionally, as the engineered oak is sold as a floating overlay floor, it is assumed that it does not cup to a problematic extent. If installed as a floating floor in situations where low humidity levels are likely, the cross-laminated floor may develop problematic levels of cupping, especially for very wide boards.

After soaking in water for two hours, all board types had cupped a similar amount, and all returned to their original shape once dry.

The engineered oak boards were significantly harder than the three types of *E. nitens* boards. Hardness has a large impact on the in-service performance of flooring, and this could cause the *E. nitens* boards to dent and damage more easily than equivalent oak boards.

Strength and stiffness varied between the different flooring types, and for the engineered oak varied between boards with a grooved back, and those without. The different *E. nitens* flooring types were at least as strong as the engineered oak boards, with some being much stronger, so it is assumed that they will have sufficient strength to perform well in service as an overlay flooring product.

Overall the key differences between the different flooring types were the lower hardness of all the *E. nitens* boards, the reduced changes in width of all the engineered flooring (*E. nitens* and oak). Additionally, for floating overlay floors, the increased cupping of the cross-laminated boards at low moisture contents may be an issue.

ACKNOWLEDGEMENTS

Graeme Manley (SouthWood Exports) donated the *E. nitens* logs.

The New Zealand Farm Forestry Association provided airfares for Dean Satchell's travelling as an in-kind contribution to this research.

Dean Satchell provided six days in-kind to sawmill and edge boards, gather data and supervise production of the products tested.

John Fairweather (Specialty Timber Solutions) produced the sample flooring product. The flooring to be tested was purchased by Scion at an agreed market rate.

Steve Chapman (Scion) assisted with preparing and labelling the boards. Jamie Agnew and Robin Parr (both Scion) assisted with measuring the boards.

John Fairweather and Dean Satchell proof read this report and provided some additional information, and suggestions for improvement.

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Standards Australia. (2006). AS 2796.2 - 2006 Timber-Hardwood-Sawn and milled products Part 2: Grade description.

APPENDIX: MATHEMATICAL BASIS FOR COMPARING DISTORTION MEASUREMENTS IN TIMBER BOARDS

1/94

Output 14252

NZ FOREST RESEARCH INSTITUTE LTD **PROJECT RECORD NO.:** 4965

DIVISION:	WOOD PROCESSING
RESOURCE CENTRE:	WOOD DRYING
CODE:	95 / 96 (Financial Year) · 403 (Resource Centre No.) · 4206 (Project) · (Sub-project) · 4415 (FRST Output) · 15 (Prog.)
WORK PLAN NO.:	
EXPERIMENT NO.:	
TITLE:	MATHEMATICAL BASIS FOR COMPARING DISTORTION MEASUREMENTS
AUTHOR(S):	STEVE RILEY
DATE:	26/10/95
KEYWORDS:	SPRING, BOW, CUP, GRADING RULES

ABSTRACT*

Basic equations for describing spring, bow, and cup in terms of radius of curvature and board length or width are derived, and a basis for comparison when length or width are changed is derived.

* **Note: This material is unpublished and must not be cited as a literature reference.**

Mathematical Basis for Comparing Distortion Measurements in Timber Boards

Steve Riley

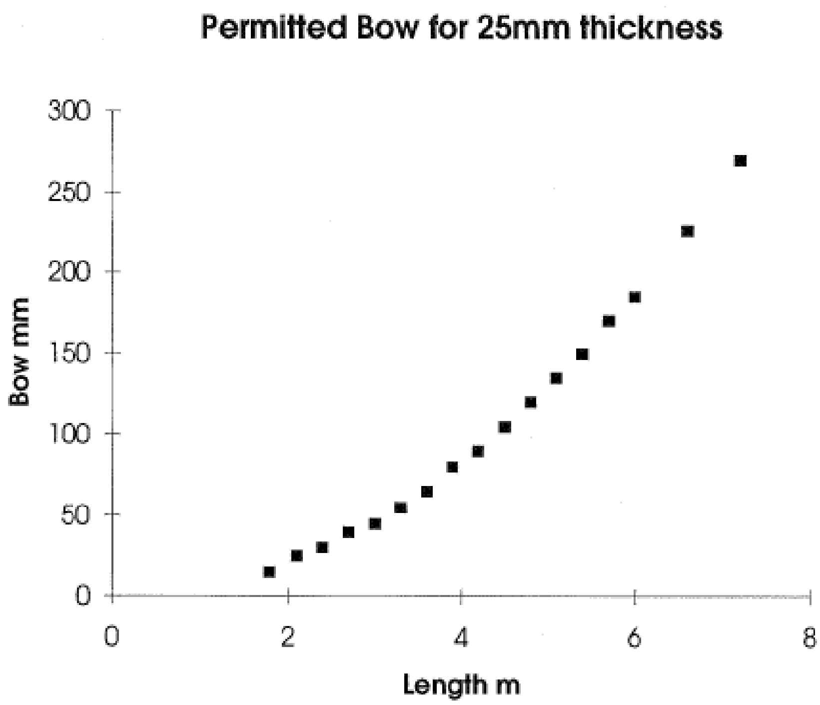
This report outlines the mathematical basis for comparing cup, spring or bow and twist between boards of different width or length. Standard timber grading rules give allowable distortion for different widths and lengths of boards, but nowhere is stated the basis for comparison or the means for interpolation or extrapolation.

I. Spring, Bow and Cup

The NZ Timber Grading Rules (1988) give the allowable bow for a 25 mm board as follows:-

Length	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	6.0	6.6	7.2
Bow	15	25	30	40	45	55	65	80	90	105	120	135	150	185	225	270

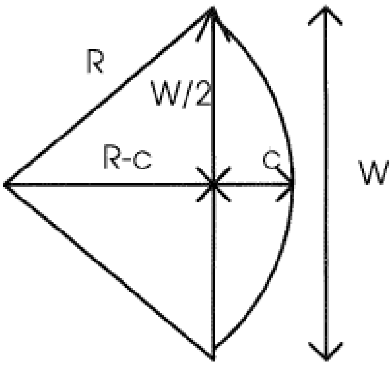
When plotted this gives an obvious parabolic shape, suggesting a squared relationship



Cup and spring also show a similar relationship. The mathematical assumptions behind all these grading rules will now be outlined.

Derivation

Cup will be used as an example. A board of width W has permitted cup of length c. Assume the cup curvature is a circle of radius R. Using simple geometry, an expression for cup length can be derived:-



Using the standard Pythagorean relationship

$(R\text{-}c)^2 = R^2 - (W/2)^2$

$R - c = \sqrt{(R^2 - (w/2)^2)}$

$c = R - \sqrt{(R^2 - (w/2)^2)}$ - eqn (1)

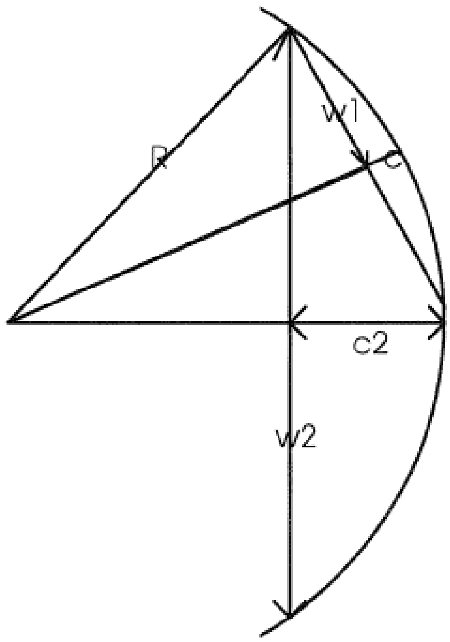
Similarly an expression for the radius of curvature :-

$(R\text{-}c)^2 + (W/2)^2 = R^2$

$R^2 - 2Rc + c^2 + w^2/2 = R^2$

$R = c^2/2 + w^2/8c$ -eqn (2)

When comparing boards of different width, say w1 and w2, assume the same radius of curvature gives cup values c1 and c2, ie :-



To relate permitted cup for boards of different width, an expression relating c1 and c2 is required. Thus using equation 1, we get :-

$$c1/c2 = \{ R - \sqrt{(R^2 - (w_1 / 2)^2)} \}/\{R - \sqrt{(R^2 - (w_2 / 2)^2)} \}$$

To simplify, use the standard binomial expansion:-

$(1 \pm x)^{½} = 1 \pm ½ .x - ½ .¼.x^2 + \ldots\ldots$ where $x \leq 1$

and expand $(R^2 - (w/2)^2)$ as $R^2(1 -(w/2R)^2)$.

Note that $w/2R \leq 1$. For example from NZ Timber Grading Rules pp 32, for w=100mm, c = 1mm and using eqn(2), R=1250.5 and w/2R= 0.04 which much less than 1.

Can now simplify the expression for c1/c2 using:-

$$\sqrt{(R^2 - (w / 2)^2)} \approx R(1 - ½(w/2R)^2)$$

Thus,

$$c1/c2 = [R -(R(1 - ½(w1/2R)^2))]/ [R -(R(1 - ½(w2/2R)^2)]$$

$$= ½(w1/2R)^2/ ½(w2/2R)^2$$

$$= (w1/w2)^2$$

or

$$c1 = c2.(w1/w2)^2$$

Thus permitted spring, bow should vary with the ratio of length squared, and cup with the ratio of width squared.

In general, for spring, bow and cup,

New permitted distortion/old permitted distortion
= (new length dimension/old length dimension)^2

This can be seen from the grading rules, allowing for the fact that measurements are rounded to the nearest 5 mm.
eg From the above table a board of length 2.4 m has a permitted bow of 30 mm. At 3.3 the above rule gives a permitted bow of $30x(3.3/2.4)^2$=56.7 mm. The table allows 55 mm, which is the above value rounded to the nearest 5mm.

II. Twist

As board length increases, it is fairly obvious and intuitive that permitted twist should increase linearly and should need no derivation. This can be seen from the table below. The first two rows are from the grading rules. Predicting permitted twist using the point at 3m as a reference and assuming

new permitted twist/old permitted twist = new length/old length

gives the third row, which becomes identical to the permitted twist when rounded to the nearest 5mm.

Length m	1.8	2.4	3	3.6	4.2	4.8	5.4	6	6.6	7.2
Permitted twist mm	10	10	15	20	20	25	25	30	35	35
Predicted twist mm	9	12	15	18	21	24	27	30	33	36

Thus:-

In general, for twist distortion in timber boards

New permitted distortion/old permitted distortion
= (new length dimension/old length dimension)

References

"New Zealand Timber Grading Rules" NZS 3631:1988 SANZ